Reasoning in Epistemic Contexts

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Conceptual Background

- In a manuscript note entitled "Towards a Science of Epistemology", John McCarthy wrote:

  > "Epistemology is about to leave philosophy and join the sciences as other disciplines have done in the past. Briefly, work on the artificial intelligence problem (i.e. the problem of making machines, specifically electronic computers, behave intelligently) will require a theory of how knowledge can be obtained by a system in interaction with its environment, what knowlege is (i.e. its mathematical structure), and what the limits of knowledge are. This is just the domain of epistemology." (McCarthy Papers, 1958-1962)

- I will take it that part of the epistemological work consists in providing a general framework for representing knowledge and exploiting knowledge by inferential means.
- I will defend that epistemological contextualism does provide such a general framework, which is also capable of expressing a plurality of concepts of knowledge.

Problem 1

- In *Contextual Computing*, Porzel (2011) underlines the first problem contextualism is facing :

  > "A general feature of context and contextual computing is lack of consensus concerning of the word itself." (9)

- Context is a blanket term that refers to some kind of dependence of an object on its environment (be it physical or conceptual).
- Among the different approaches in AI, for which the notion of context is of primary importance, one finds knowledge representation and natural language processing.
- My proposal will take place in the knowledge representation approach.
- One of the main challenges in this regard lies in defining a rigorous notion of context, i.e. a notion workable in a formal system.

Problem 2

- The second problem that needs to be addressed is clearly put into light by Guha and McCarthy (2003):

  > "If the statements ϕ and $\phi \Rightarrow \beta$ have different contextual dependencies, the program [inference engine] can't always combine them to conclude β. Before combining two sentences with different contextual dependencies, the program needs to reconcile relative contextual dependencies." (166)

- In the formal representation, not only the contextual dependencies should be made explicit, but the *relation* between contexts should be qualified in order to allow, or not, for inferences.

  > "To cope with the contextuality, it [the program] needs to be able to factor out the relative contextuality so that it can use knowledge gathered in one context in another." (106)

- This problem presents two different aspects: a *logical* aspect, which refers to formal representation, and an *epistemological* aspect, which refers to the epistemic qualification of contextual dependencies.

CL_{MCB}

- The path I will follow is the one opened up by McCarthy (1993), and developed by Guha (1991, 2003) and Buvač (1995), which consists in treating a context as a formal object embedded in logic.
- Buvač wrote:

  > "The logic of context provides a language in which contexts are treated as formal objects. The logic extends classical first-order logic by introducing a modality, $\mathtt{ist}(c, \phi)$, used to express that the formula, ϕ, holds in context c." (1995, 102)

- The operator `ist` is a formal tool for disambiguating formulas that contain ambiguous expressions. Disambiguation is thus understood as the assignment of a value to a formula in function of a context.
- The contextual logic of McCarthy and Buvač (CL_{MCB}) can be defined as $FOL \cup \{\mathtt{ist}(c, \phi)\}$, where $\mathtt{ist}(c, \phi)$ is an operator meaning that the formula ϕ is true in context c.

CL_{MCB}

- For example, consider the following two formulas specifying the meaning of *Holmes* in different contexts (McCarthy 1993):

  $c0$: ist(context-of("Sherlock Holmes stories"), "Holmes is a detective")
  $c0$: ist(context-of("U.S. legal history"), "Holmes is a Court Justice")

- McCarthy's view on context allows for operations on contexts: *entering* and *exiting* (or *lifting*) a context.
- About *entering* a context, McCarthy (1993) wrote:

  > "Suppose we have the sentence $\mathtt{ist}(c, p)$. We can then enter the context c and infer the sentence p. We can regard $\mathtt{ist}(c, p)$ as analogous to $c \supset p$, and the operation of entering c as analogous to assuming c in a system of natural deduction as invented by Gentzen and described in many logic texts."

- When *exiting* a context with respect to a particular formula, the operator `ist` is used to indicate the context of origin. For instance, if a formula p can be asserted in a context c, then the outer context $c0$ includes the formula $\mathtt{ist}(c, p)$.

Some Properties of Contexts in CL_{MCB}

K: $\vdash_k \mathtt{ist}(k', \phi \supset \psi) \supset (\mathtt{ist}(k', \phi) \supset \mathtt{ist}(k', \psi))$

Δ: $\vdash_k \mathtt{ist}(k_1, \mathtt{ist}(k_2, \phi) \vee \psi) \supset \mathtt{ist}(k_1, \mathtt{ist}(k_2, \phi)) \vee \mathtt{ist}(k_1, \psi)$

Flat: $\vdash_k \mathtt{ist}(k_2, \mathtt{ist}(k_1, \phi)) \supset \mathtt{ist}(k_1, \phi)$

Enter: $\dfrac{\vdash_{k'} \mathtt{ist}(k, \phi)}{\vdash_k \phi}$

Exit: $\dfrac{\vdash_k \phi}{\vdash_{k'} \mathtt{ist}(k, \phi)}$

BF: $\vdash_k (\forall v)\mathtt{ist}(k', \phi) \supset \mathtt{ist}(k', (\forall v)\phi)$

Epistemological Perspective

- From an epistemological perspective, CL_{MCB} yields two significant conceptual gains:
  1. The contextual dependencies are formalized by means of an operator.
  2. This operator can act as an indexical term (Guha and McCarthy 2003).
- These aspects are perfectly in line with the core thesis of epistemological contextualism, which is based upon an indexical interpretation of the knowledge predicate (or knowledge operator).
- In a formalism like CL_{MCB}, it is possible to set explicitely the properties of each epistemic context, and to explore (by inferential means) the variety of intracontextual and intercontextual relations among epistemic contexts.

Indexical interpretation of K

- The `ist` operator can serve as a basis for an understanding of the knowledge operator (K-operator) interpreted indexically.
- In the proposed framework, and following the usual distinction from Kaplan, the indexical meaning of the K-operator will be:

  Content means the epistemic standard (ε) that governs a context.

  Character means the assertability of a formula in a given context (in accordance with a given ε).

- The indexical content of K is precisely given by ε. The invariable part of the meaning of K makes it a *success term*.
- In this perspective, the notion of epistemic context can be given priority and can be at the center the epistemological investigation.

Epistemic Context

- An epistemic context c is a context defined by an epistemic standard ε that specifies the introduction rule for the knowledge operator in c (while providing its indexical content).
- The complete specification of an epistemic context depends on a twofold characterization:
  1. A characterization of its *epistemic standard* ε, and
  2. A characterization of its *transfer rules* τ (if any), which are the rules that govern the relations with other epistemic contexts and that account for context shifts.
- In defining epistemic contexts by means of explicit epistemic standards, one not only gives the knowledge operator its various meanings, but one also describes a structure into which epistemic normativity is spelled out in different terms.

Epistemological Theory

- Such a conception allows for multiple configurations of epistemic contexts, which in turn can be captured by the idea that an epistemological theory is a *set of epistemic contexts.*
- For instance, consider an epistemological theory Θ defined by three epistemic contexts, say $C_{log}, C_{emp}, C_{per}$. Then, Θ would include three epistemic standards, $\varepsilon_{log}, \varepsilon_{emp}, \varepsilon_{per}$, and possibly some transfer rules, $\tau_{log}, \tau_{emp}, \tau_{per}$.
- An epistemological theory is consequently defined by a specific set of epistemic contexts (or knowledge bases) that include among their axioms epistemic standards and transfer rules.
- Foundationalism, coherentism, reliabilism, and other options based on the JTB model, may be construed as exemplifying different epistemological structures designed to meet different epistemic demands.

Zoo Example

- Let us confront these ideas with Dretske's Zebra Case, according to which a visitor at a zoo, who presumably knows that the animals in the pen marked "Zebras" are zebras, cannot know that these animals are really zebras since she does not know whether these animals are cleverly disguised mules.
- We need first to define an epistemological theory comprising three epistemic contexts $\{C_{log}, C_{emp}, C_{per}\}$:

  ε_{log} $(\forall x)(\phi \supset K(x, \phi))$,
  where ϕ is a tautology or a valid first-order formula.

  ε_{emp} $(\forall x)(\mathit{EmpiricalControl}(x, \phi) \supset K(x, \phi))$

  ε_{per} $(\forall x \forall v)((\mathit{See}(x, v) \vee \mathit{Hear}(x, v) \vee \mathit{Taste}(x, v) \vee \mathit{Smell}(x, v) \vee \mathit{Touch}(x, v)) \supset K(x, \phi))$,
  where ϕ is in immediate relation to v.

Zoo Example

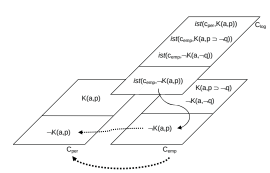
- The epistemic facts are:
  1. $\vdash_{c_{per}} K(a, p)$ *premise*
  2. $\vdash_{c_{emp}} K(a, p \supset \neg q)$ *premise*
  3. $\vdash_{c_{emp}} \neg K(a, \neg q)$ *premise*
- In order to use logic on this information, one must first apply the *Exit* rule into the logical context:
  4. $\vdash_{c_{log}} ist(c_{per}, K(a, p))$ *Exit*, 1
  5. $\vdash_{c_{log}} ist(c_{emp}, K(a, p \supset \neg q))$ *Exit*, 2
  6. $\vdash_{c_{log}} ist(c_{emp}, \neg K(a, \neg q))$ *Exit*, 3

Zoo Example

- By applying logical closure, contraposition, and *modus ponens* one obtains:
  5. $\vdash_{c_{log}} ist(c_{emp}, K(a, p \supset \neg q))$ *Exit*, 2
  6. $\vdash_{c_{log}} ist(c_{emp}, \neg K(a, \neg q))$ *Exit*, 3
  7. $\vdash_{c_{log}} ist(c_{emp}, K(a, p)) \supset ist(c_{emp}, K(a, \neg q))$ Closure, 5
  8. $\vdash_{c_{log}} ist(c_{emp}, \neg K(a, \neg q)) \supset ist(c_{emp}, \neg K(a, p))$ Contrapo., 7
  9. $\vdash_{c_{log}} ist(c_{emp}, \neg K(a, p))$ Modus Ponens 6,8
  10. $\vdash_{c_{emp}} \neg K(a, p)$ *Enter*, 9
  11. $\vdash_{c_{per}} \neg K(a, p)$?
- Assertion 11 exhibits the critical move, that relies upon a transfer from $\vdash_{c_{emp}} \neg K(a, p)$ to $\vdash_{c_{per}} \neg K(a, p)$.

Zoo Example

- The skeptical strategy can be diagrammed in this way (where the dotted lines represent the litigious transfers):



Corollaries

- The contextualist thesis is fundamentally a hybrid thesis, incorporating a partial invariantism, inasmuch as it relies upon an indexical interpretation of the knowledge predicate.
- Epistemological contextualism presents a major conceptual gain: it is a theory about the *normative function* of epistemic standards. In this respect, epistemological contextualism presents itself as a general *epistemological framework.*
- From the contextualist point of view, epistemic normativity is a normative function distributed and realized in a plurality of spaces whose dimensionalities are defined by different epistemic standards.

Requisites

- The proposed ND system satisfies a number of constraints, among which:
  1. K is an operator.
  2. Epistemic standards are encapsulated in contexts.
  3. Relations between contexts are explicit in the system.
  4. The knowledge introduction rule does not allow for the construction of formulas of type $K_iK_i\Phi$.
- As with the `ist` operator, the K-operator must preserve the distinction between the *satisfaction* of an epistemic standard and the *standard* itself.

ND Rules

Reiteration (R)

$\vdots$	$\vdots$	
m	Φ	
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	
n	Φ	R, m

Any hypothesis can be reiterated in the same context. This is a restricted form of the usual ND rule.

ND Rules

Knowledge reiteration (KR)

$\vdots$		$\vdots$	
m		$K_i\Phi$	
$\vdots$		$\vdots$	
$\vdots$		$\vdots$	
n	i	$\vdots$	
o		$K_i\Phi$	KR, m

A hypothesis of type $K_i\Phi$ can be reiterated in a K-context i.

ND Rules

Knowledge elimination (KE)

$\vdots$	i	$\vdots$	
m		$K_i\Phi$	
$\vdots$		$\vdots$	
n		Φ	KE, m

This conforms with the principle of factivity of knowledge.

ND Rules

Knowledge introduction (KI)

1		Φ_1	
$\vdots$		$\vdots$	
m		Φ_m	
$\vdots$		$\vdots$	
n	i	$\vdots$	
o		Ψ	
$o+1$		$K_i\Psi$	KI, n–o

provided Ψ is not prefixed by K_i.

This expresses the idea that the formula Ψ satisfies the epistemic standard which defines the K-context i. The rule prohibits the construction of formulas of type $K_iK_i\Psi$.

ND Rules

Negation of knowledge introduction (¬KI)

1		Φ_1	
$\vdots$		$\vdots$	
m		Φ_m	
$\vdots$		$\vdots$	
n	i	Ψ	
$\vdots$		$\vdots$	
o		Ω	
$\vdots$		$\vdots$	
p		$\neg\Omega$	
$p+1$		$\neg K_i\Psi$	¬KI, n–p

In order to negate the knowledge of a formula Ψ in a K-context i, one has to provide a proof *ad absurdum* under the hypothesis Ψ.

ND Rules

Knowledge transfer (KT)

$\vdots$		$\vdots$	
m		$K_i\Phi$	
$\vdots$		$\vdots$	
n	j	$\vdots$	
o		$K_j\Phi$	KT_{i-j}, m

A formula of type $K_i\Phi$ can be transferred in a K-context j, where $i \neq j$, by means of a transfer rule $i-j$ such that KT_{i-j} : *if* $K_i\Phi$ *then* $K_j\Phi$. The KT rule is a qualified reiteration rule through epistemic contexts, on the basis of a conditional epistemic qualification.

Remarks

- In this framework, the notion of context is a syntactical notion.
- The rule KE expresses the idea that the information encapsulated by the K-operator is available only within the associated epistemic context.
- The rule KI captures the idea that when the information from a context is viewed from another context, the information of the former context should be encapsulated so that what is made available to the latter context is a reference (index or pointer) to the contextual origin of the information.
- Transfer rules can be added as hypotheses.
- The MCB rules *Enter* and *Exit* correspond respectively, in ND-CEL, to KR+KE and KI, and the `ist` operator is rendered by the indexical character of the K-operator (satisfaction of the assertability conditions).

Formulas of ND-CEL

- Here are some derivable formulas:
  1. $K(p \wedge q) \vdash Kp \wedge Kq$
  2. $Kp \wedge Kq \vdash K(p \wedge q)$
  3. $K(p \supset q) \vdash Kp \supset Kq$
  4. $K(p \equiv q) \vdash Kp \equiv Kq$
  5. $\{Kp, K(p \supset q)\} \vdash Kq$
  6. $K_1p \vdash K_2K_1p$
- In the literature, some of these formulas are litigious from an epistemological point of view, *closure* (5) and *introspection* (6) in particular, but they remain in line with contextualism.

Transfer Rules

- In addition to the epistemic standards, the transfer rules contribute significantly to specify an epistemological theory.
- For instance, consider the following ND-CEL systems w/r to transfer rules related to the main logical context:

$\varnothing$	KT_{i-0}	KT_{0-i}
$Kp \nvdash p$	$Kp \vdash p$	$Kp \nvdash p$
$p \nvdash Kp$	$p \nvdash Kp$	$p \vdash Kp$
$\{Kp, p \supset q\} \nvdash Kq$	$\{Kp, p \supset q\} \vdash Kq$	$\{Kp, p \supset q\} \vdash Kq$
$\nvdash Kp \supset p$	$\nvdash Kp \supset p$	$\vdash Kp \supset p$
$\nvdash p \supset Kp$	$\nvdash p \supset Kp$	$\vdash p \supset Kp$

KK-Thesis

- The first ligitious formula is the KK-thesis. Its proof in ND-CEL is straightforward:

1			K_1p	
2	*2*	*1*	K_1p	KR, 1
3			p	KE, 2
4			K_1p	KI, 2–3
5			K_2K_1p	KI, 2–4
6			$K_1p \supset K_2K_1p$	⊃I, 1–5

- This result is instructive about contextualism itself, because it shows the *character* of the indexical interpretation of knowledge, i.e., when one knows, one also knows *that* some epistemic standard has been satisfied.

KK-Thesis

- On the other hand, when knowledge is interpreted *univocally* the KK-thesis cannot hold in ND-CEL:

1		K_1p	
2	*1*	K_1p	KR, 1
3		K_1K_1p	? (KI is not permitted)
4		$K_1p \supset K_1K_1p$	⊃I, 1–3

- Such an interpretation is typical of epistemological theories that rely on a concept of knowledge understood in representational terms (cognition).
- For instance, in the case of a perception, an univocal schema of the thesis implies a perceptual impossibility, as long as one cannot see herself seeing a table: $See^a_n(table) \supset See^a_n(See^a_n(table))$.

Epistemic Closure

- The second ligitious formula is the closure principle. Again, its proof in ND-CEL is quite direct:

1		K_1p	
2		$K_1(p \supset q)$	
3	*1*	K_1p	KR, 1
4		p	KE, 3
5		$K_1(p \supset q)$	KR, 2
6		$p \supset q$	KE, 5
7		q	⊃E, 4, 6
8		K_1q	KI, 3–7

- The deduction is valid because the K-context remains constant throughout the deduction.

Epistemic Closure

- But in the Zoo example, there is a context shift that requires a transfer rule, since Kp and $K(p \supset \neg q)$ cannot be both assertable in a perceptual context:

1		K_1p	
2		$K_2(p \supset \neg q)$	
3	*1*	K_1p	KR, 1
4		p	KE, 3
5		$K_1(p \supset \neg q)$	? (requires KT_{2-1})
6		$p \supset \neg q$	KE, 5
7		$\neg q$	⊃E, 4, 6
8		$K_1\neg q$	KI, 3–7

- In order to have a valid conclusion, one needs an epistemological theory that allows for such a knowledge transfer.
- The closure principle, by itself, does not provide a transfer rule.

Conclusion

- CL_{MCB} can be used as a formal resource for analyzing knowledge under an indexical interpretation.
- CL_{MCB} offers a perspective on epistemic normativity that does not appeal to the notion of epistemic justification.
- Contextualism is a theory about the normative function of epistemic standards. In this respect, epistemological contextualism is a general *epistemological framework*.
- The problem of context shifting receives a direct solution insofar all contextual changes are regulated by transfer rules that proceed from the epistemic standards defining the contexts, in accordance to a given epistemological theory.

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