The Phylogeny and Ontogeny of Deductive Reasoning: A Cultural Account

Catarina Dutilh Novaes
Faculty of Philosophy – University of Groningen

Plan of the talk

1. Phylogeny: the dialogical origins of deductive reasoning
2. Deductive proof as a dialogical notion
3. Ontogeny: learning to play the game of deduction

Introduction

What is the nature of deductive reasoning? (Understood as necessary truth-preservation)
Is it 'in our heads' in some innate way, or does it require special training to be learned and performed?
Claim: deductive reasoning is a cultural phenomenon, tightly connected to dialogical practices. This is the case both for 'phylogeny' and for 'ontogeny'. (Dutilh Novaes 2013)

Phylogeny and ontogeny of mathematical reasoning

"The history of mathematics and the logic of mathematical discovery, i.e. the phylogenesis and the ontogenesis of mathematical thought…" (Lakatos, *Proofs and Refutations*, p. 4)

"Zoologists maintain that the embryonic development of an animal recapitulates in brief the whole history of its ancestors throughout geologic time. It seems it is the same in the development of minds… For this reason, the history of science should be our first guide." (Poincaré, *The Foundations of Science*, p. 437)

1. Phylogeny: the dialogical origins of deductive reasoning

Ancient Greece: dialectical practices in the early Academy (Castelnerac & Marion 2009).
Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating. Syllogistic is less obviously about debating, but the *Prior Analytics* is full of dialectical/dialogical references.
Latin Middle Ages: *logica = dialectica*

Adversariality and cooperation

In a debate, the two parties are disputing: adversariality. But besides the element of adversariality, influence of the Socratic didactic model. (Dutilh Novaes 2015)

Opponent must also become *convinced* of the conclusion; the argumentation must be perspicuous and explanatory.

> "Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs ... But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern... It is at once oral and written..." (Netz 1999, 297/8)

2. Deductive proof as a dialogical notion

- What is the point of deductive proofs? What are they good for? (Rav 1999; Auslander 2008)
- The main function of a deductive proof is to produce explanatory persuasion in an interlocutor (Hersh 1993). A good proof is one that convinces a fair but 'stubborn' opponent of the truth of a given statement, given the (presumed) truth of other statements.
- So proof is an inherently dialogical, multi-agent notion (Ernest 1994).

Rational reconstruction
Proofs as semi-adversarial dialogues of a special kind. The participants have opposite goals:

- **Prover**: establishing the conclusion.
- **Skeptic**: blocking the establishment of the conclusion.

The dialogue starts with Prover asking Skeptic to grant certain premises. Prover puts forward further statements, which purportedly follow from what has been granted.
Skeptic's moves: to present counterexample and 'why does it follow?' question.

Every proof is a discourse aimed at a putative audience.

- Persuasiveness: a proof must be perspicuous for Skeptic
- Necessary truth-preservation: a proof must be indefeasible, winning strategy for Prover
- Policies for premise acceptance: goes back to dialogical practices of proposing, granting etc.

Internalization of Skeptic
The job of Skeptic is to look for counterexamples and to make sure the argumentation is perspicuous. But the deductive method has internalized Skeptic by integrating these features:

- The strategic goal of formulating indefeasible arguments becomes a constitutive feature of the method itself: necessary truth-preservation.
- The didactic goal of formulating convincing arguments becomes the ideal of explanatory, self-evident proofs.

3. Ontogeny: learning to play the game of deduction

- The reasoner must be prepared to reason with premises regardless of her beliefs about them.
- Deductive arguments must be truth-preserving: monotonicity.
- Argument has to be broken down in small, individually perspicuous steps.

Claim: all these features are 'cognitive oddities' which require special training to be mastered.

Accepting premises

It may seem that reasoning with premises one does not believe in is an unproblematic, spontaneous cognitive task.

However, reasoning studies with unschooled participants show that this too appears to be a skill that must be learned. Unschooled participants typically refuse to reason with premises they have no knowledge of.

> "In the Far North, where there is snow, all bears are white. Novaya Zemlya is in the Far North. What colour are bears there?" In response to this problem, [a given participant] protested: "You've seen them – you know. I haven't seen them, so how could I say!?" [...] the interviewer encouraged him to focus on the wording of the problem: "But on the basis of what I said, what do you think?" and re-stated the problem. This repetition met with the same refusal: "But I never saw them, so how could I say?" (Harris 2000, 96)

However, schooled participants also 'struggle' (Oakhill & Johnson-Laird 1985):

> Some of the actresses are not beautiful.
> All of the women are beautiful.
>
> Some of the A are not B
> All of the C are B
> Thus, some of the A are not C

Some of the actresses are not women (correct)	38%
No valid conclusion (error)	46%
Other errors	16%

Necessary truth-preservation: indefeasibility

Experiments with deductive reasoning tasks have shown time and again that participants (including highly schooled ones) deviate considerably from the canons of deductive reasoning. (Evans 2002)

Arguably, the main discrepancy pertains to the concept of indefeasibility: spontaneous human reasoning is highly defeasible (Oaksford & Chater 2002; Stenning & van Lambalgen 2008).

Is monotonic, deductive reasoning something that can be *learned*? (Morris & Sloutsky 1998)

There is consensus in the mathematics education literature that students at all levels tend to have difficulties producing and understanding proofs (Robert & Schwarzenberger 1991).

Emulating the game of deduction

The claim is that the basic features of deductive reasoning (indefeasibility, dissociation from belief, perspicuity) have dialogical origins.

Can we exploit this idea to teach deductive reasoning more efficiently? (Does ontogeny recapitulate phylogeny?)

- Accepting premises 'for the sake of the argument'
- Indefeasibility = adversariality
- Perspicuity: explanation, convincing a peer

Cueing acceptance of premises

P. Harris and collaborators (Dias & Harris 1988; Dias et at. 2005): with different experimental setup, participants are much more willing to reason with arbitrary premises, and performance in experiments comes much closer to the deductively 'correct' responses.

Premises are presented differently:

- Children: in the form of story telling.
- Adults: as about 'another planet'.

Cueing indefeasible arguments

Simulating adversariality: betting. Achourioti and Stenning conducted a pilot study with undergraduates: formulation of syllogistic reasoning tasks as bets (Stenning 2012).

Participant plays against 'Harry the Snake', and is given fully formulated syllogistic arguments.

- If conclusion follows, she should not bet.
- If not, she bets against Harry and must provide a counterexample.

Preliminary results: large numbers of participants shift towards classical (indefeasible) responses.

Cueing perspicuity and persuasiveness

Work by Alibert, Balacheff etc. has focused on the social dimension of proof, as a tool to advance debates among peers.

"By establishing an environment in which students may see and experience first-hand what is necessary for them to *convince others* of the truth or falsehood of propositions, proof becomes an instrument of personal value which they will be happier to use (or teach) in the future." (Alibert & Thomas, 1991, 230)

Groups outperform individuals in deductive tasks

In a number of experiments, it has been shown that groups achieve the 'correct answer' much more often than individuals.

Wason selection task: 10% correct by individuals, 70-80% correct by groups (same population) (Moshman & Geil 1998).

Individuals with the best answers are able to convince others through argumentation (Mercier & Sperber 2011).
It has also been shown that individuals are better at evaluating the validity of arguments than at producing valid arguments (Trouche et al. 2014).

4. Conclusions

• Deduction is best viewed as a dialogical notion.
• The main features of deductive reasoning are 'cognitive oddities' whose emergence are best accounted for as cultural phenomena.
- Taking premises at face value
- Indefeasible arguments
- Small, perspicuous inferential steps (the least 'odd').

- Phylogeny: dialectical practices in Ancient Greece.
- Ontogeny: dialogical cues facilitate deductive reasoning performance.

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