Reasoning with Epistemic Modals

Handout

Malte Willer

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Institut des sciences cognitives
Université du Québec à Montréal
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1 Key Scenario

1. Mary is in Chicago or in New York, but not both. $C\dot{\vee}NY$
2. Mary might be in Chicago, and she might be in New York. $\Diamond C \wedge \Diamond NY$
3. If Mary is not in Chicago, then she must be in New York. $(\text{if } \neg C)(\Box NY)$
4. If Mary is not in New York, then she must be in Chicago. $(\text{if } \neg NY)(\Box C)$
5. Mary cannot be in Chicago and in New York. $\neg \Diamond (C \wedge NY)$

2 Observations and Classical Assumptions

- (1)–(5) are consistent.
- If $\Sigma \vDash (\text{if } \phi)(\psi)$, then $\Sigma, \phi \vDash \psi$
- **Duality**: $\Diamond\phi =_{\text{def}} \neg \Box \neg\phi$
- **Reductio**: if $\Sigma, \phi \vDash \bot$, then $\Sigma \vDash \neg\phi$
- **Monotonicity**: if $\Sigma \vDash \psi$, then $\Sigma, \phi \vDash \psi$
- **Distribution**: if $\Sigma \vDash \Box(\phi \supset \psi)$ and $\Sigma \vDash \Box\phi$, then $\Sigma \vDash \Box\psi$
- **Necessitation**: if $\vDash \phi$, then $\vDash \Box\phi$

3 Update Semantics

- A possible world w is a function from atomic sentences to truth-values.
- An information state s is a set of possible worlds; a state s is consistent just in case it is non-empty.
- Basic update rules:
  1. $s + P = s \cap [[P]]$
  2. $s + \neg\phi = s \smallsetminus (s + \phi)$
  3. $s + (\phi \wedge \psi) = (s + \phi) + \psi$

4. $s + \Diamond\phi = \begin{cases} s & \text{if } s + \phi \neq \varnothing \\ \varnothing & \text{otherwise} \end{cases}$

5. $s + \Box\phi = \begin{cases} s & \text{if } s + \phi = s \\ \varnothing & \text{otherwise} \end{cases}$

- Further definitions:
  - s is *comitted* to ϕ iff $s + \phi = s$
  - *Entailment*: $\phi_1, \dots, \phi_n \vDash \psi$ iff for all s: $s + \phi_1 + \dots + \phi_n$ is committed to ψ
  - $\phi_1, \dots, \phi_n$ is *consistent* iff for some s: $s + \phi_1 + \dots + \phi_n \neq \varnothing$
  - $\phi_1, \dots, \phi_n$ is coherent iff for some $s \neq \varnothing$: s is *committed* to ϕ_1, …, s is committed to ϕ_n
- Update rules for conditionals:
  - $s + (\text{if } \phi)(\psi) = \begin{cases} s & \text{if } s + \phi + \psi = s + \phi \\ \varnothing & \text{otherwise} \end{cases}$
- A few facts:
  - $\Diamond P \vDash \Diamond P$ but $\Diamond P, \neg P \nvDash \Diamond P$, so US '$\vDash$' is nonmonotonic.
  - Dynamic conditionals support modus ponens.
  - (1)–(5) are consistent.
  - $\Diamond P \wedge \neg P$ is consistent, $\neg P \wedge \Diamond P$ is inconsistent (but both fail to be coherent)